Attendance Problems. Write each fraction as a decimal rounded to the nearest hundredths.

1. $\frac{2}{3}$ **2.** $\frac{7}{24}$

Solve each equation.

3. $0.8 = \frac{5.8}{x}$ **4.** $0.94 = \frac{x}{8.5}$

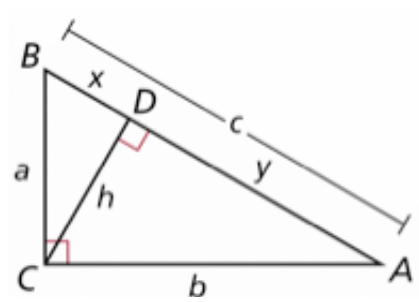
- I can find the sine, cosine, and tangent of an acute angle.
- I can use trigonometric ratios to find side lengths in right triangles and to solve real-world problems.
- I can use the relationship between the sine and cosine of complementary angles.

Vocabulary	
trigonometric ratio	sine
cosine	tangent

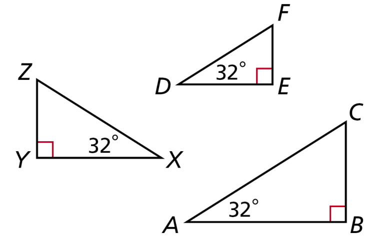
<u>Common Core</u>

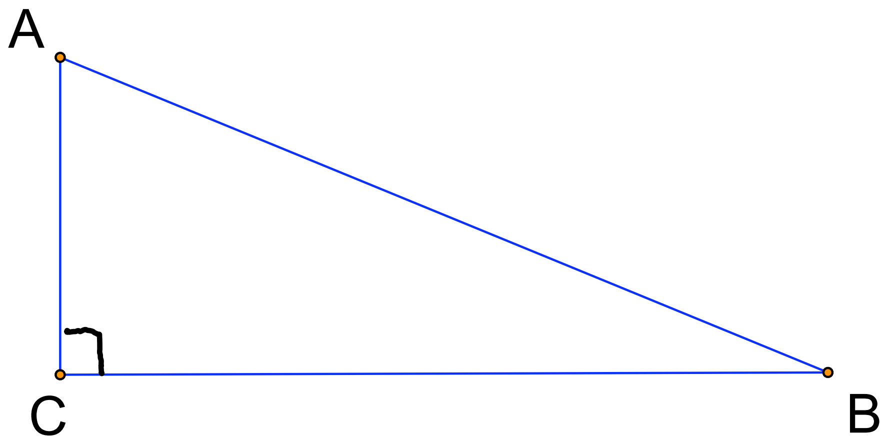
CC.9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CC.9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

5. Prove the Pythagorean Theorem.



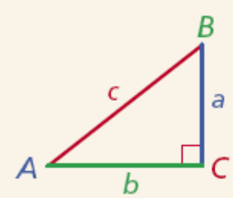
By the AA Similarity Postulate, a right triangle with a given acute angle is similar to every other right triangle with that same acute angle measure. So $\Delta ABC \sim \Delta DEF \sim \Delta XYZ$, and $\frac{BC}{AC} = \frac{EF}{DF} = \frac{YZ}{XZ}$. These are *trigonometric ratios*. A **<u>trigonometric ratio</u>** is a ratio of two sides of a right triangle.





Acute Angle	Opposite Side	Adjacent Side	Hypotenuse

Trigonometric Ratios

DEFINITION	SYMBOLS	DIAGRAM
The **sine** of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{a}{c}$ $\sin B = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{b}{c}$	
The **cosine** of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{b}{c}$ $\cos B = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{a}{c}$	
The **tangent** of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{a}{b}$ $\tan B = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{b}{a}$	

Q: What happened to the geometry student who went to the beach to catch some rays?
A: He became a tangent

SOHCAHTOA!

(Handel's Hallelujah Chorus)

Soh Cah Toa!
Soh Cah Toa!
Learn it, and use it!
Soh Cah Toa!

Sine is opposite over hypotenuse.
Soh Cah Toa
Soh Cah Toa
Learn it, and use it!

Cosine is adjacent over hypotenuse.
Soh Cah Toa
Soh Cah Toa
Learn it, and use it!

Tangent is opposite over adjacent!
Soh Cah Toa
Soh Cah Toa

SOH CAH TOA!

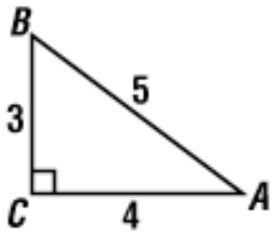
Writing Math

In trigonometry, the letter of the vertex of the angle is often used to represent the measure of that angle. For example, the sine of $<A$ is written as $\sin A$.

"Wise sayings often fall on barren ground, but a kind word is never thrown away."—*19th Century Writer, Arthur Helps*

Refer to video Example 1.

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



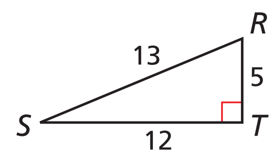
sin A

cos A

tan B

1 Finding Trigonometric Ratios

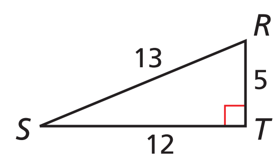
Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



A $\sin R$

$\sin R = \frac{12}{13} \approx 0.92$ *The sine of an ∠ is* $\frac{\text{opp. leg}}{\text{hyp.}}$.

Write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



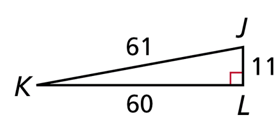
B $\cos R$

$\cos R = \frac{5}{13} \approx 0.38$ *The cosine of an ∠ is* $\frac{\text{adj. leg}}{\text{hyp.}}$.

C $\tan S$

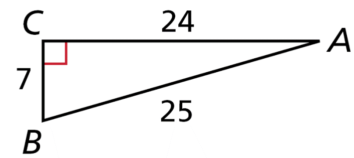
$\tan S = \frac{5}{12} \approx 0.42$ *The tangent of an ∠ is* $\frac{\text{opp. leg}}{\text{adj. leg}}$.

Example 1. Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest ten-thousandth.



A. sin J **B.** cos J **C.** tan K

<u>Guided Practice</u>. Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest ten-thousandth.



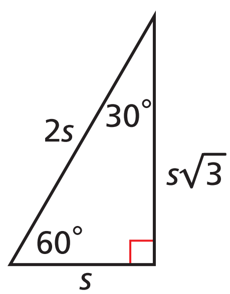
6. cos A **7.** tan B **8.** sin B

<u>Refer to videoExample 2</u>.

Use a special right triangle to write tan 30° as a fraction.

2 Finding Trigonometric Ratios in Special Right Triangles

Use a special right triangle to write sin 60° as a fraction.



Draw and label a 30°-60°-90° △.

$$\sin 60° = \frac{s\sqrt{3}}{2s} = \frac{\sqrt{3}}{2}$$

The sine of an ∠ is $\frac{\text{opp. leg}}{\text{hyp.}}$.

<u>Example 2</u>. Use a special right triangle to write cos 30° as a fraction.

9. <u>Guided Practice</u>. Use a special right triangle to write tan 45° as a fraction.

<u>Video Example 3</u>. Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

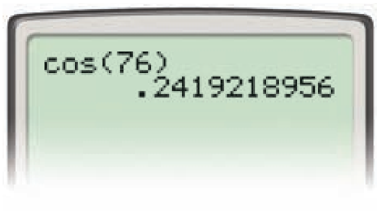
A. cos 51° **B.** sin 29° **C.** tan 78°

Caution!
Be sure your calculator is in degree mode, not radian mode.

3 Calculating Trigonometric Ratios

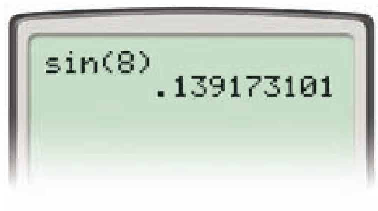
Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

A **cos 76°**



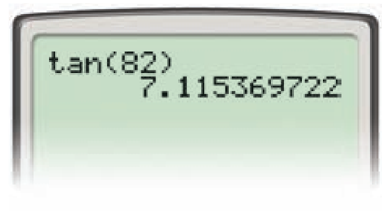
cos 76° ≈ 0.24

B **sin 8°**



sin 8° ≈ 0.14

C **tan 82°**



tan 82° ≈ 7.12

<u>Example 3</u>. Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

A. sin 52° **B.** cos 19° **C.** tan 65°

<u>Guided Practice</u>. Use your calculator to find the trigonometric ratio. Round to the nearest ten-thousandth.

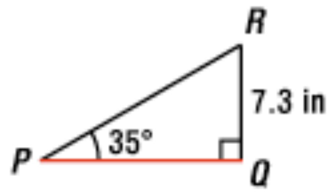
10. tan 11° **11.** sin 62° **12.** cos 30°

<u>8-2 Trigonometric Ratios:</u> (pp 545) 23, 27, 29, 30, 33, 35.

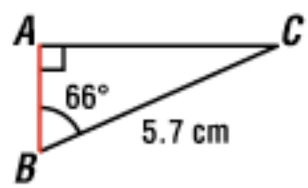
The hypotenuse is always the longest side of a right triangle. So the denominator of a sine or cosine ratio is always greater than the numerator. Therefore the sine and cosine of an acute angle are always positive numbers less than 1. Since the tangent of an acute angle is the ratio of the lengths of the legs, it can have any value greater than 0.

<u>Refer to video example 4</u>. Find the length. Round to the nearest hundredth.

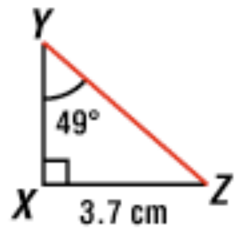
PQ



AB

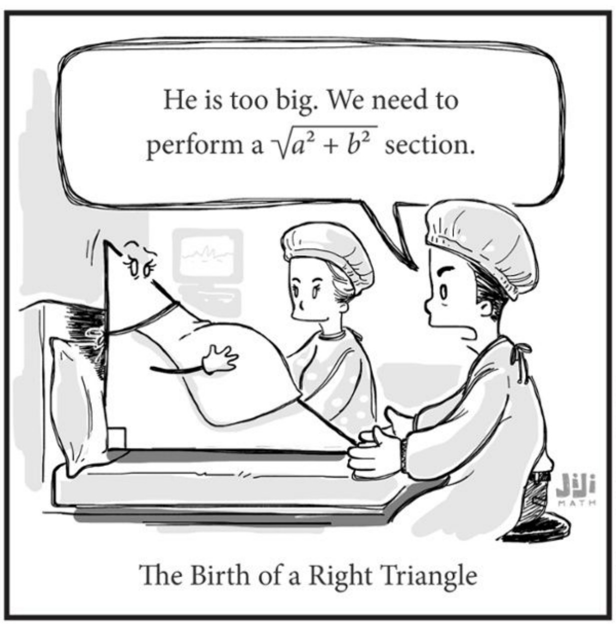


YZ



Caution!

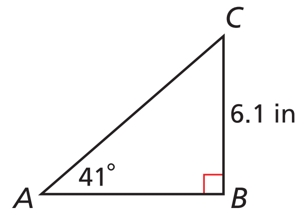
Do not round until the final step of your answer. Use the values of the trigonometric ratios provided by your calculator.



4 Using Trigonometric Ratios to Find Lengths

Find each length. Round to the nearest hundredth.

A *AB*



$\overline{AB}$ is adjacent to the given angle, $\angle A$. You are given BC, which is opposite $\angle A$. Since the adjacent and opposite legs are involved, use a tangent ratio.

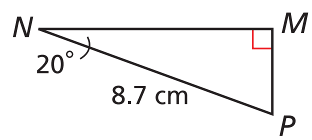
$\tan A = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{BC}{AB}$ *Write a trigonometric ratio.*

$\tan 41° = \frac{6.1}{AB}$ *Substitute the given values.*

$AB = \frac{6.1}{\tan 41°}$ *Multiply both sides by AB and divide by tan 41°.*

$AB \approx 7.02$ in. *Simplify the expression.*

B *MP*



$\overline{MP}$ is opposite the given angle, $\angle N$. You are given NP, which is the hypotenuse. Since the opposite side and hypotenuse are involved, use a sine ratio.

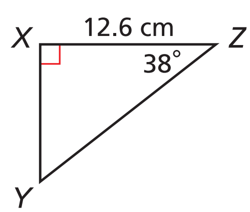
$\sin N = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{MP}{NP}$ *Write a trigonometric ratio.*

$\sin 20° = \frac{MP}{8.7}$ *Substitute the given values.*

$8.7(\sin 20°) = MP$ *Multiply both sides by 8.7.*

$MP \approx 2.98$ cm *Simplify the expression.*

C *YZ*



YZ is the hypotenuse. You are given XZ, which is adjacent to the given angle, $\angle Z$. Since the adjacent side and hypotenuse are involved, use a cosine ratio.

$\cos Z = \frac{\text{adj. leg}}{\text{hyp.}} = \frac{XZ}{YZ}$ *Write a trigonometric ratio.*

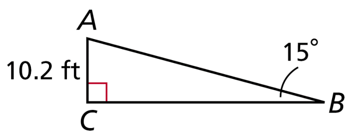
$\cos 38° = \frac{12.6}{YZ}$ *Substitute the given values.*

$YZ = \frac{12.6}{\cos 38°}$ *Multiply both sides by YZ and divide by cos 38°.*

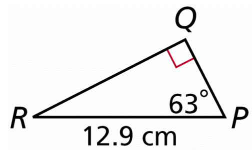
$YZ \approx 15.99$ cm *Simplify the expression.*

<u>Example 4</u>. Find the length. Round to the nearest tenth.

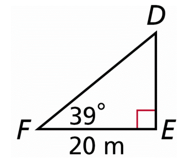
A. BC



B. QR

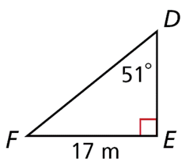


C. FD

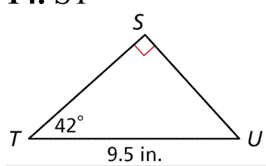


<u>Guided Practice</u>. Find the length. Round to the nearest tenth.

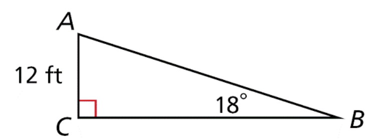
13. DF



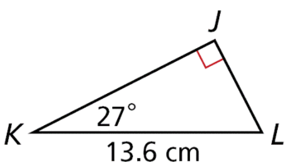
14. ST



15. BC

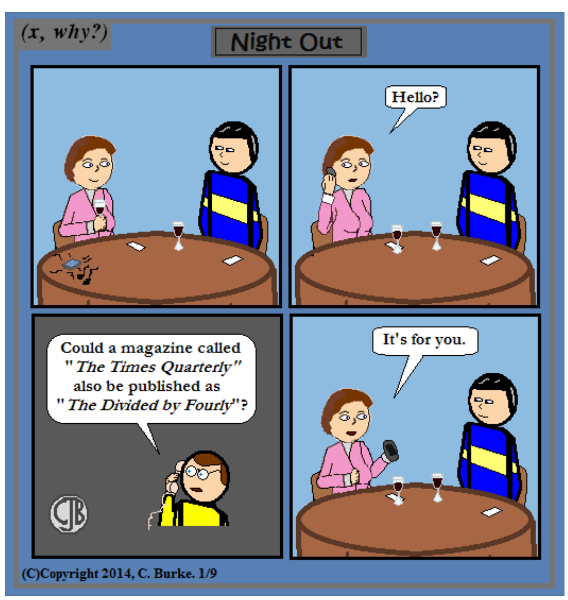


16. JL



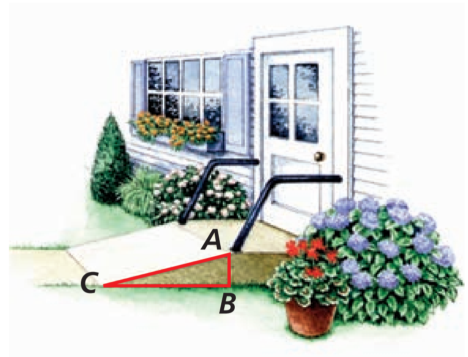
<u>Refer to video Example 5</u>.

A contractor is building a wheelchair ramp for a doorway that is 1.6 ft above the ground. To meet ADA guidelines, the ramp will make an angle of 4.1° with the ground. To the nearest hundredth of a foot, what is the horizontal distance covered by the ramp?



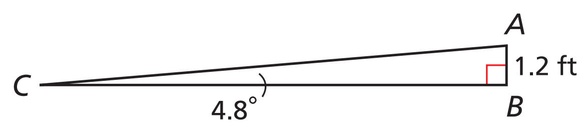
5 Problem Solving Application

A contractor is building a wheelchair ramp for a doorway that is 1.2 ft above the ground. To meet ADA guidelines, the ramp will make an angle of 4.8° with the ground. To the nearest hundredth of a foot, what is the horizontal distance covered by the ramp?



1 Understand the Problem

Make a sketch. The **answer** is *BC*.



2 Make a Plan

$\overline{BC}$ is the leg adjacent to $\angle C$. You are given *AB*, which is the leg opposite $\angle C$. Since the opposite and adjacent legs are involved, write an equation using the tangent ratio.

3 Solve

$\tan C = \dfrac{AB}{BC}$ — *Write a trigonometric ratio.*

$\tan 4.8° = \dfrac{1.2}{BC}$ — *Substitute the given values.*

$BC = \dfrac{1.2}{\tan 4.8°}$ — *Multiply both sides by BC and divide by tan 4.8°.*

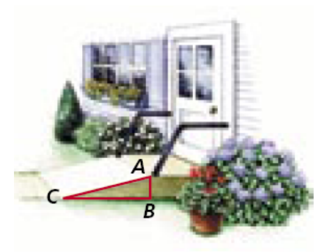
$BC \approx 14.2904$ ft — *Simplify the expression.*

4 Look Back

The problem asks for *BC* rounded to the nearest hundredth, so round the length to 14.29. The ramp covers a horizontal distance of 14.29 ft.

<u>Example 5</u>. The Pilatusbahn in Switzerland is the world's steepest cog railway. Its steepest section makes an angle of about 25.6° with the horizontal and rises about 0.9 km. To the nearest hundredth of a kilometer, how long is this section of the railway track?

17. <u>Guided Practice</u>. A contractor is building a wheelchair ramp for a doorway that is 1.2 ft above the ground. To meet ADA guidelines, the ramp will make an angle of 4.8° with the ground. Find *AC*, the length of the ramp, to the nearest hundredth of a foot.



<u>8-2 Trigonometric Ratios</u>: (pp 545) 23, 27, 29, 30, 33, 35, 39, 41, 43, 48, 50, 64, 66, 67.

The acute angles of a right triangle are complementary angles. If the measure of one of the two acute angles is given, the measure of the second acute angle can be found by subtracting the given measure from 90°.

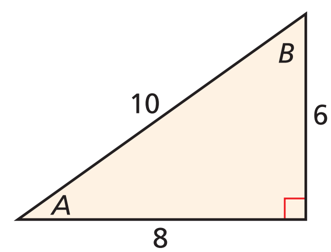
1 Finding the Sine and Cosine of Acute Angles

Find the sine and cosine of the acute angles in the right triangle shown.

Start with the sine and cosine of ∠*A*.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$



Then, find the sine and cosine of ∠*B*.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

18. <u>Guided Practice</u>. Find the sine and cosine of the acute angles of a right triangle with sides 10, 24, 26. (Use A for the angle opposite the side with length 10 and B for the angle opposite the side with length 24.)

The trigonometric function of the complement of an angle is called a ***cofunction***. The sine and cosines are cofunctions of each other.

2 Writing Sine in Cosine Terms and Cosine in Sine Terms

A Write sin 42° in terms of the cosine.

$$\sin 42° = \cos(90 - 42)°$$
$$= \cos 48°$$

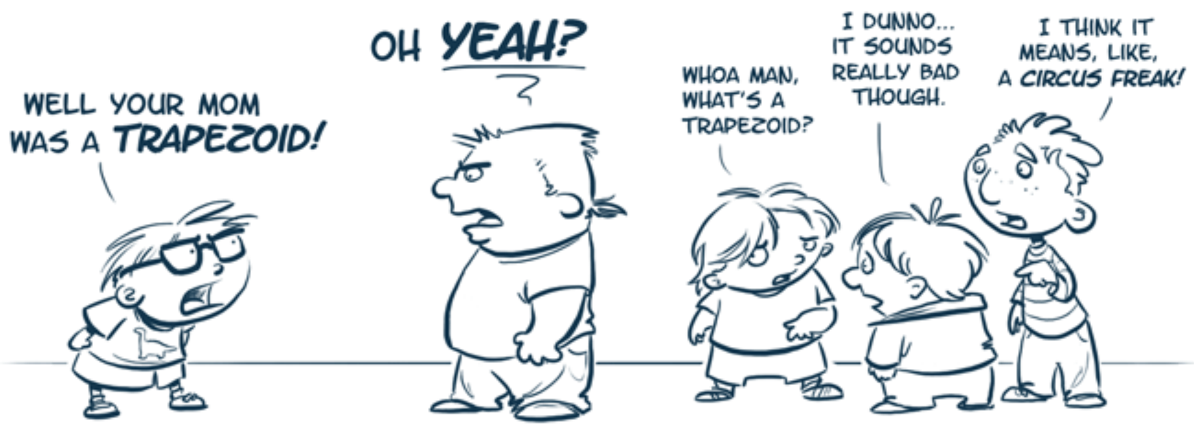
B Write cos 36° in terms of the sine.

$$\cos 36° = \sin(90 - 36)°$$
$$= \sin 54°$$

Guided Practice.

19. Write sin 28° in terms of the cosine.

20. Write cos 51° in terms of the sine.



Timmy's Advanced Geometry class did come in handy at times.

3 Finding Unknown Angles

Find two angles that satisfy the equation.

$$\sin(2x - 4)° = \cos(3x + 9)°$$

If $\sin(2x - 4)° = \cos(3x + 9)°$, then $(2x - 4)°$ and $(3x + 9)°$ are the measures of complementary angles. The sum of the measures must be 90°.

$$\begin{aligned} (2x - 4) + (3x + 9) &= 90 \\ 5x + 5 &= 90 \\ 5x &= 85 \\ x &= 17 \end{aligned}$$

Substitute the value of x into the original expression to find the angle measures.

$$\begin{aligned} 2x - 4 &= 2(17) - 4 \\ &= 30° \end{aligned}$$

$$\begin{aligned} 3x + 9 &= 3(17) + 9 \\ &= 60° \end{aligned}$$

The measurements of the two angles are 30° and 60°.

Guided Practice.

21. $\sin(3x + 2)° = \cos(x + 44)°$

22. $\sin(2x + 20)° = \cos(3x + 30)°$

8-2 Trigonometric Ratios

- (pp 545) 23, 27, 29, 30, 33, 35, 39, 41, 43, 48, 50, 64, 66, 67.
- (p 550) 1, 5, 7, 13, 15.

"I've decided to forego trigonometry, and make myself eligible for the NBA draft."